Chapter 14: Dual Nature of Radiation and Matter

EXERCISES [PAGES 322 - 323]

Exercises | Q 1.1 | Page 322

Choose the correct option.
A photocell is used to automatically switch on the street lights in the evening when the sunlight is low in intensity. Thus it has to work with visible light. The material of the cathode of the photocell is

1. Zinc
2. aluminum
3. nickel
4. **potassium**

SOLUTION

potassium

Exercises | Q 1.2 | Page 322

Choose the correct option.
Polychromatic (containing many different frequencies) radiation is used in an experiment on the photoelectric effect. The stopping potential

1. will depend on the average wavelength
2. will depend on the longest wavelength
3. **will depend on the shortest wavelength**
4. does not depend on the wavelength

SOLUTION

will depend on the shortest wavelength

Exercises | Q 1.3 | Page 322

Choose the correct option.
An electron, a proton, an α-particle, and a hydrogen atom are moving with the same kinetic energy. The associated de Broglie wavelength will be longest for

1. **electron**
2. proton
3. α-particle
4. hydrogen atom

SOLUTION

electron

Exercises | Q 1.4 | Page 322

Choose the correct option.

If N_{Red} and N_{Blue} are the numbers of photons emitted by the respective sources of equal power and equal dimensions in unit time, then

1. $N_{Red} < N_{Blue}$
2. $N_{Red} = N_{Blue}$
3. **$N_{Red} > N_{Blue}$**
4. $N_{Red} \approx N_{Blue}$

SOLUTION

$N_{Red} > N_{Blue}$

Exercises | Q 1.5 | Page 322

Choose the correct option.

The equation E = pc is valid

1. for all sub-atomic particles
2. is valid for an electron but not for a photon
3. **is valid for a photon but not for an electron**
4. is valid for both an electron and a photon

SOLUTION

Is valid for a photon but not for an electron

Exercises | Q 2.1 | Page 322

Answer in brief.

What is the photoelectric effect?

SOLUTION

The phenomenon of emission of electrons from a metal surface when electromagnetic radiation of appropriate frequency is incident on it is known as photoelectric effect.

Exercises | Q 2.2 | Page 322

Answer in brief.

Can microwaves be used in the experiment on photoelectric effect?

SOLUTION

No.

Explanation:

The photoelectric effect is totally a Quantum Physics phenomenon. A single photon of light delivers it's a tiny bundle of energy to the metal and ejects a single electron with a very precise energy balance.

Electric sparks emitted from metal points/edges in a microwave oven are a macroscopic phenomenon that has no need for a quantum explanation. The microwave generator pumps hundreds of watts of electromagnetic energy into the oven cavity. The microwaves have a frequency of 2.4 gigahertz which gives them a wavelength of about 12 cm. (About 4 3/4 inches for those in the US)

These intense electric fields induce currents to flow in the metal. Sharp points/edges concentrate the buildup of the electrons which can locally increase the voltage enough to cause the arcing.

Exercises | Q 2.3 | Page 322

Answer in brief.

Is it always possible to see the photoelectric effect with a red light?

SOLUTION

No, it is not possible but due to present technology it may be possible

Explanation:

The photons in red light to not have the necessary energy required to rip an electron out of its orbital (this needed energy is equal to the electrons "work function). Because light behaves like particles rather than a continuous stream, even very high -intensity red light will never be able to overcome an electrons' work function (in this situation), as every individual photon fails to do so. This shows the particle behavior of light, because of light behaved like a wave, the red light would be able to overcome the electron's work function with high intensity or a long time.

Exercises | Q 2.4 | Page 322

Using the values of work function given in the following table, tell which metal will require the highest frequency of incident radiation to generate photocurrent. Typical values of work function for some common metals

Metal	Work function (in eV)
Potassium	2.3
Sodium	2.4
Calcium	2.9
Zinc	3.6
Silver	4.3
Aluminium	4.3
Tungsten	4.5
Copper	4.7

Nickel	5.0
Gold	5.1

SOLUTION

Gold is the highest frequency of incident radiation to generate photocurrent.

Exercises | Q 2.5 | Page 322

What do you understand by the term wave-particle duality? Where does it apply?

SOLUTION

Depending upon experimental conditions or the structure of matter, electromagnetic radiation and material particles exhibit wave nature or particle nature. This is known as wave-particle duality.
It applies to all phenomena. The wave nature and particle nature are liked by the de Broglie relation λ = h/p, where λ is the wavelength of matter waves, also called de Broglie waves I Schr6dinger waves, p is the magnitude of the momentum of a particle or quantum of radiation and h is the universal constant called Planck's constant.

Exercises | Q 3 | Page 322

Explain the inverse linear dependence of stopping potential on the incident wavelength in a photoelectric effect experiment.

SOLUTION

We have $V_0 e = \frac{hc}{\lambda} - \phi$, where V_0 is the stopping potential, e is the magnitude of the charge on the electron, his Planck's constant, c is the speed of light in free space, λ is the wavelength of the electromagnetic radiation incident on a metal surface and Φ is the work function for the metal, h, c and e are constants. Φ is constant for a particular metal.

Hence, it follows that as $\frac{1}{\lambda}$ increases, V_0 increases.

The plot of V_0 verses $\frac{1}{\lambda}$ is linear. This is because the energy associated with a quantum of radiation (photon) is directly proportional to the frequency of the radiation and hence inversely proportional to the wavelength of radiation.

Exercises | Q 4 | Page 322

It is observed in an experiment on the photoelectric effect that an increase in the intensity of the incident radiation does not change the maximum kinetic energy of the electrons. Where does the extra energy of the incident radiation go? Is it lost? State your answer with explanatory reasoning.

SOLUTION

When electromagnetic radiation with a frequency greater than the threshold frequency is incident on a metal surface, there is an emission of electrons. It is observed that not every incident photon is effective in liberating an electron. In fact, the number of electrons emitted per second is far less than the number of photons incident per second. The photons that are not effective in the liberation of electron s are reflected (or scattered) or absorbed resulting in rising in the temperature of the metal surface. The maximum kinetic energy of a photoelectron depends on the frequency of the incident radiation and the threshold frequency for the metal. It has nothing to do with the intensity of the incident radiation. The increase in intensity results in an increase in the number of electrons emitted per second.

Exercises | Q 5 | Page 322

Explain what you understand by the de Broglie wavelength of an electron. Will an electron at rest have an associated de Broglie wavelength? Justify your answer.

SOLUTION

Under certain conditions, an electron exhibits wave nature. Waves associated with a moving electron are called matter waves or de Broglie waves orSchr6dinger waves. The de Broglie wavelength of these matter waves is given by).. = h Ip, where h is Planck's constant, and p is the magnitude of the momentum of the electron.

If an electron is at rest, its momentum would be zero, and hence the corresponding de Broglie wavelength would be the infinite indicating the absence of a matter-wave. However, according to quantum mechanics/wave mechanics, this is not possible.

Exercises | Q 6 | Page 322

State the importance of Davisson and Germer experiment.

SOLUTION

The Davisson and Germer experiment are probably one of the most important experiments ever since it verified that De Broglie's “matter wave” hypothesis applied to matter (electrons) as well as light. From this emerged modern quantum theory, the most stupendous revolution in physics of all time.

Exercises | Q 7 | Page 322

What will be the energy of each photon in monochromatic light of frequency 5×10^{14}Hz?

SOLUTION

Data: $\nu = 5 \times 10^{14}$ Hz, $h = 6.63 \times 10^{-34}$ J-s, $1 eV = 1.6 \times 10^{-19}$ J

The energy of each photon,

$E = h\nu = (6.63 \times 10^{-34} \text{ J-s})(5 \times 10^{14} \text{ Hz})$

$= 3.315 \times 10^{-19}$ J

$$= \frac{3.315 \times 10^{-19} \mathrm{J}}{1.6 \times 10^{-19} \mathrm{J/eV}}$$

$= 2.072$ eV

Exercises | Q 8 | Page 323

Observations from an experiment on the photoelectric effect for the stopping potential by varying the incident frequency were plotted. The slope of the linear curve was found to be approximately 4.1×10^{-15} V s. Given that Exercises the charge of an electron is 1.6×10^{-19} C, find the value of the Planck's constant h.

SOLUTION

Data: Slope $= 4.1 \times 10^{-15}$ V·s, $e = 1.6 \times 10^{-19}$ C $V_0 e = h\nu - h\nu_0$

$$\therefore V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{h\nu_0}{e}\right)$$

$$\therefore \text{Slope} = \frac{h}{e}$$

∴ Planck's constant,

$h = (\text{slope})(e) = (4.1 \times 10^{-15} \text{ V·s})(1.6 \times 10^{-19} \text{ C})$

$$= 6.56 \times 10^{-34} \text{ J.s} \left(\text{as } 1 \text{ V} = \frac{1\text{J}}{1\text{C}}\right)$$

Exercises | Q 9 | Page 323

The threshold wavelength of tungsten is 2.76×10^{-5} cm.
(a) Explain why no photoelectrons are emitted when the wavelength is more than 2.76×10^{-5} cm.

(b) What will be the maximum kinetic energy of electrons ejected in each of the following cases
(i) if ultraviolet radiation of wavelength $\lambda = 1.80 \times 10^{-5}$ cm and
(ii) radiation of frequency 4×10^{15} Hz is made incident on the tungsten surface?

SOLUTION

Data: $\lambda_0 = 2.76 \times 10^{-5}$ cm $= 2.76 \times 10^{-7}$ m,
$\lambda = 1.80 \times 10^{-5}$ cm $= 1.80 \times 10^{-7}$ m,
$v = 4 \times 10^{15}$ Hz,
$h = 6.63 \times 10^{-34}$ J.s,
$c = 3 \times 10^{8}$ m/s

(a) For $\lambda > \lambda_0, v < v_0$ (threshold frequency).

$\therefore hv < hv_0$. Hence, no photoelectrons are emitted.

(b) Maximum kinetic energy of electrons ejected

$$= \mathbf{hc}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= \left(6.63 \times 10^{-34}\right)\left(3 \times 10^{8}\right)\left(\frac{10^{7}}{1.8} - \frac{10^{7}}{2.76}\right) \text{J}$$

$= (6.63 \times 10^{-19})(0.5555 - 0.3623)$

$= (6.63)(0.1932 \times 10^{-19})\text{J} = 1.281 \times 10^{-19}\text{J}$

$$= \frac{1.281 \times 10^{-19}\text{J}}{1.6 \times 10^{-19}\text{J/eV}}$$

$= 0.8006$ eV

(c) Maximum kinetic energy of electrons ejected

$$= hv - \frac{\mathbf{hc}}{\lambda_0}$$

$$= \left(6.63 \times 10^{-34}\right)\left(4 \times 10^{15}\right) - \frac{\left(6.63 \times 10^{-34}\right)\left(3 \times 10^{8}\right)}{2.76 \times 10^{-7}}$$

$= 26.52 \times 10^{-19} - 7.207 \times 10^{-19}$

$= 19.313 \times 10^{-19}$ J

$$= \frac{19.313 \times 10^{19} \text{J}}{1.6 \times 10^{-19} \text{J/eV}}$$

= 12.07 eV

Exercises | Q 10 | Page 323

Photocurrent recorded in the microammeter in an experimental setup of the photoelectric effect vanishes when the retarding potential is more than 0.8 V if the wavelength of incident radiation is 4950 Å. If the source of incident radiation is changed, the stopping potential turns out to be 1.2 V. Find the work function of the cathode material and the wavelength of the second source.

SOLUTION

Data: V_O = 0.8 V, λ = 4950 Å = 4.950 x 10^{-1} m, $V_{O'}$ = 1.2 V, h = 6.63 × 10^{-34} J.s, c = 3 × 10^8 m/s

1. $V_O e = hv - \phi = \frac{hc}{\lambda} - \phi$

∴ The work function of the cathode material,

$$\phi = \frac{hc}{\lambda} - V_O e$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4.950 \times 10^{-7}} - (0.8)(1.6 \times 10^{-19})$$

= 4.018 × 10^{-19} - 1.28 × 10^{-19} = 2.738 × 10^{-19} J

$$= \frac{2.738 \times 10^{-19} \text{J}}{1.6 \times 10^{-19} \text{J/eV}} = 1.711 \text{ eV}$$

2. $V_{O'} e = \frac{hc}{\lambda'} - \phi$

$$\therefore \frac{hc}{\lambda'} = V_{O'} e + \phi$$

∴ The wavelength of the second source,

$$\lambda' = \frac{hc}{V_{O'} e + \phi}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(1.2)(1.6 \times 10^{-19}) + 2.738 \times 10^{-19}}$$

$$= \frac{19.89 \times 10^{26}}{4.658 \times 10^{-19}}$$

$= 4.270 \times 10^{-7}$ m = 4270 Å

Exercises | Q 11 | Page 323

Radiation of wavelength 4500 Å is incident on a metal having work function 2.0 eV. Due to the presence of a magnetic field B, the most energetic photoelectrons emitted in a direction perpendicular to the field move along a circular path of radius 20 cm. What is the value of the magnetic field B?

SOLUTION

Data: λ = 4500 Å = 4.5×10^{-7} m,

Φ = 2.0 eV = $2 \times 1.6 \times 10^{-19}$ J,

h = 6.63×10^{-34} J.s, c = 3×10^{8} m/s,

r = 20 cm = 0.2 m, e = 1.6×10^{-19} C, m = 9.1×10^{-31} kg

$$\frac{1}{2}mv_{max}^{2}(KE_{max}) = \frac{hc}{\lambda} - \cdot\phi$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{4.5 \times 10^{-7}} - 3.2 \times 10^{-19}$$

$= 4.42 \times 10^{-19} - 3.2 \times 10^{-19}$

$= 1.22 \times 10^{-19}$ J

$$\therefore v_{max} = \sqrt{\frac{2}{m}KE_{max}} = \sqrt{\frac{2 \times 1.22 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= \sqrt{0.2681 \times 10^{12}} = 5.178 \times 10^{5} \text{ m/s}$$

Now, centripetal force, $\frac{mv_{max}^{2}}{r}$ = magnetic force, Bev_{max}

$$\therefore B = \frac{mv_{max}}{re} = \frac{(9.1 \times 10^{-31})(5.178 \times 10^5)}{(0.2)(1.6 \times 10^{-19})}$$

$= 1.472 \times 10^{-5}$ T

Exercises | Q 12 | Page 323

Given the following data for incident wavelength and the stopping potential obtained from an experiment on the photoelectric effect, estimate the value of Planck's constant and the work function of the cathode material. What is the threshold frequency and corresponding wavelength? What is the most likely metal used for emitter?

Incident wavelength (in Å)	2536	3650
Stopping potential (in V)	1.95	0.5

SOLUTION

Data: $\lambda = 2536$ Å $= 2.536 \times 10^{-7}$ m,

$\lambda' = 3650$ Å $= 3.650 \times 10^{-7}$ m,

$V_0 = 1.95$ V, $V_0' = 0.5$ V, $c = 3 \times 10^8$ m/s,

$e = 1.6 \times 10^{-19}$ C

$$\textbf{(i)}\ V_0 e = \frac{hc}{\lambda} - \phi \text{ and } V_0\prime e = \frac{hc}{\lambda\prime} - \phi$$

$$\therefore (V_0 - V_0)e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda\prime}\right)$$

$\therefore (1.95 - 0.5)(1.6 \times 10^{-19})$

$$= h\,(3 \times 10^8)\left(\frac{10^7}{2.536} - \frac{10^7}{3.650}\right)$$

$\therefore 2.32 \times 10^{-19} = h(3 \times 10^{15})(0.3943 - 0.2740)$

$\therefore$ 2.32 × 10^{-19} = h(3 × 10^{15})(0.3943 - 0.2740)

$\therefore$ h = $\frac{2.32 \times 10^{34}}{0.3609} = 6.428 \times 10 - 34$ J.s

This is the value of Planck's constant.

(ii) $\phi = \frac{hc}{\lambda} - V_0 e$

$= \frac{(6.428 \times 10^{-34})(3 \times 10^8)}{2.536 \times 10^{-7}} - (1.95)(1.6 \times 10^{-19})$

$= 7.604 \times 10^{-19} - 3.12 \times 10^{-19} = 4.484 \times 10^{-19}$ J

$= \frac{4.484 \times 10^{-19} J}{1.6 \times 10^{-19} J/eV}$

= 2.803 eV is the work function of the cathode material.

(iii) Φ = hv_0

$\therefore$ The threshold frequency, $v_0 = \frac{\phi}{h}$

$= \frac{4.484 \times 10^{-19} J}{6.428 \times 10^{-34} J.s} = 6.976 \times 10^{14}$ Hz

(iv) $v_0 = \frac{c}{\lambda_0}$

$\therefore$ The threshold wavelength, $\lambda_0 = \frac{c}{v_0}$

$= \frac{3 \times 10^8}{6.976 \times 10^{14}} = 4.300 \times 10^{-7}$ = 4300 Å

(v) The most likely metal used for emitter: calcium.

Exercises | Q 13.1 | Page 323

Calculate the wavelength associated with an electron, its momentum and speed when it is accelerated through a potential of 54 V.

SOLUTION

Data: V = 54 V, m = 9.1 x 10^{-31} kg, e = 1.6 x 10^{-19}C, h = 6.63 x 10^{-34} J.s, KE = 150 eV

We assume that the electron is initially at rest.

$\therefore$ Ve = $\frac{1}{2}mv^2$

$\therefore$ v = $\sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2(54)\left(1.6 \times 10^{-19}\right)}{9.1 \times 10^{-31}}}$

$= \sqrt{19 \times 10^{12}}$ = 4.359 × 10^6 m/s

This is the speed of the electron.

p = mv = (9.1 × 10^{-31})(4.359 × 10^6)

= 3.967 × 10^{-24} kg.m/s

This is the momentum of the electron. The wavelength associated with the electron,

λ = $\frac{h}{p} = \frac{6.63 \times 10^{-34}}{3.967 \times 10^{-24}} = 1.671 \times 10^{-10}$m

= 1.671 Å = 0.1671 nm

Exercises | Q 13.2 | Page 323

Calculate the wavelength associated with an electron, its momentum and speed when it is moving with the kinetic energy of 150 eV.

SOLUTION

Data: V = 54 V, m = 9.1 x 10-31 kg,

e = 1.6 x 10-19C, h = 6.63 x 10-34 J.s, KE = 150 eV

As KE ∝ $\sqrt{V}$, we get

$\frac{v'}{v} = \sqrt{\frac{150}{54}}$ = 1.666

$\therefore$ v' = 1.666 v = (1.666)(4.359 × 10^6)

= 7.262 × 10^6 m/s

This is the speed of the electron.

p' = mv' = $(9.1 \times 10^{-31})(7.262 \times 10^{6})$

= 6.608×10^{-24} kg.m/s

This is the momentum of the electron. The wavelength associated with the electron,

$$\lambda\prime = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6.608 \times 10^{-24}} = 1.003 \times 10^{-10}\text{m}$$

= 1.003 Å = 0.1003 nm

Exercises | Q 14 | Page 323

The de Broglie wavelengths associated with an electron and a proton are the same. What will be the ratio of (i) their momenta (ii) their kinetic energies?

SOLUTION

Data: λ (electron)= λ (proton)

m (proton)= 1836 m (electron)

(i) λ = $\frac{h}{p}$ As λ (electron) = λ (proton),

$\frac{p(\text{electron})}{p(\text{proton})} = 1$, where p denotes the magnitude of momentum.

(ii) Assuming v « c,

KE = $\frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$

$\frac{KE(\text{electron})}{KE(\text{proton})} = \frac{m(\text{proton})}{m(\text{electron})}$ = 1836 as p is the same for the electron and the proton.

Exercises | Q 15 | Page 323

Two particles have the same de Broglie wavelength and one is moving four times as fast as the other. If the slower particle is an α-particle, what are the possibilities for the other particle?

SOLUTION

Data: $\lambda_1 = \lambda_2$, $v_1 = 4v_2$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\therefore \lambda_1 = \frac{h}{m_1 v_1}, \lambda_2 = \frac{h}{m_2 v_2}$$

As $\lambda_1 = \lambda_2$, $m_1 v_1 = m_2 v_2$

$$\therefore m_1 = m_2 \frac{v_2}{v_1} = m_2\left(\frac{1}{4}\right) = \frac{m_2}{4}$$

As particle 2 is the ex-particle, particle 1 (having the mass 1/4 times that of the a-particle) may be a proton or neutron.

Exercises | Q 16 | Page 323

What is the speed of a proton having de Broglie wavelength of 0.08 Å?

SOLUTION

Data: λ = 0.08 Å = 8×10^{-12} m, h = 6.63×10^{-34} J.s, m = 1.672×10^{-27} kg

$$\lambda = \frac{h}{mv}$$

$$\therefore v = \frac{h}{m\lambda}$$

$$v = \frac{6.63 \times 10^{-34}}{\left(8 \times 10^{-12}\right)\left(1.672 \times 10^{-27}\right)}$$

$v = 4.957 \times 10^4$ m/s

Exercises | Q 17 | Page 323

In nuclear reactors, neutrons travel with energies of 5×10^{-21} J. Find their speed and wavelength.

SOLUTION

Data: KE = 5 × 10^{-21} J, m = 1.675 × 10^{-27} kg, h = 6.63 × 10^{-34} J.s

KE = $\frac{1}{2}mv^2 = 5 \times 10^{-21}$ J

$\therefore$ v = $\sqrt{\frac{2KE}{m}} = \sqrt{\frac{(2)(5 \times 10^{-21})}{1.675 \times 10^{-27}}}$

= 2.443 × 10^3 m/s

This is the speed of the neutrons. The de Broglie wavelength associated with the neutron,

λ = $\frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(1.675 \times 10^{-27})(2.443 \times 10^3)}$

= 1.620 × 10^{-10} m = 1.620 Å

Exercises | Q 18 | Page 323

Find the ratio of the de Broglie wavelengths of an electron and a proton when both are moving with the (a) same speed, (b) the same kinetic energy, and (c) the same momentum. State which of the two will have a longer wavelength in each case.

SOLUTION

Data: m_p = 1836 m_e

(a) The de Broglie wavelength, λ = $\frac{h}{p} = \frac{h}{mv}$

$\frac{\lambda_e}{\lambda_p} = \left(\frac{m_p}{m_e}\right)\left(\frac{v_p}{v_e}\right)$ = 1836 as $v_p = v_e$

Thus, $\lambda_e < \lambda_p$.

(b) λ = $\frac{h}{p} = \frac{h}{\sqrt{2mK}}$, where K denotes the kinetic energy $\left(\frac{1}{2}mv^2\right)$

$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_pK_p}{m_eK_e}} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836} = 42.85$

as $K_p = K_e$

Thus, $\lambda_e > \lambda_p$.

(c) $\lambda = \dfrac{h}{p}$

$\therefore \dfrac{\lambda_e}{\lambda_p} = \dfrac{p_p}{p_e} = 1$ as $p_p = p_e$.